處 理 技 術

# THE MOST PROBABLE NUMBER (MPN) OF TOTAL BACTERIAL COUNT OF A WATER SAMPLE

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In water quality laboratory or in biological laboratory, the total bacterial count of a water sample is frequently expressed as the most probable number, MPN per ml of the of the sample (or per 100 mls).

The MPN is a mathematical estimation, not a statistical estimation. It is estimated from the laboratory testing results of the sample under the following conditions:

(1) The size (ml) of sample used,
(2) The number of samples tested,
(3) The number of samples showing positive (+) presence of bacteria.

The method used is the Poisson's series for computing the bacterial density, in number per unit volume (ml) of the sample.
If S=ml. of the sample size,

m=number of tubes showing presence (+) of bacteria,
n=number of tubes showing negative (−)
N=m+n=total number of tubes tested.
$\lambda$=Bacterial density in number of bacteria per ml.
$\gamma$=Total number of bacteria detected

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Then, the probability (chance) that *no* bacteria is in the given sample is $e^{-\gamma}=e^{-S\lambda}$, $\gamma$ being the total number of bacteria in the sample.

The probability (chance) that *some* bacteria are present in the given sample $=1-e^{-S\lambda}$.

For a test system showing $S=0.01^{(-)}$, $0.1^{(-)}$, $1.0^{(-)}$, $10.0^{(+)}$ 100 mls$^{(+)}$ results, we have the total probability (chance) y to be as follows:

$$Y=(1-e^{-100\lambda})\,(1-e^{-10\lambda})e^{-\lambda}e^{-0.01\lambda}$$

$$=(1-e^{-100\lambda})(1-e^{-10\lambda})e^{-1.11\lambda} \qquad (1)$$

Where the signs have the following meanigs:

(+) denotes "positive" or "present" of bacteria;

(−) denotes "negative" or "absent" of bacteria.

Then we find the MPN by differentiation of y with respect to $\lambda$ and $(dy/d\lambda)=0$.

The value of $\lambda$ is then calculated. In our case, the value of $\lambda$ mode is found to be 0.23 (MPN). The median value can be calculated, and also the mean value. We have found the median $\lambda$ value is 0.72/ml.; and the mean $\lambda$ value is 0.99/ml.

For $\lambda=0.23$ per ml, the MPN is 23/100 mls.

It must be understood that MPN can be objectionable because it may occur at the maximal point of the relation of $Y=f(\lambda)$, and it can be at the minimal point!

Thus, MPN can give falsely high or falsely low bacterial density.[1] It is perhaps necessary to review MPN with other values. The mean value of $\lambda$ is determined by the following relation:

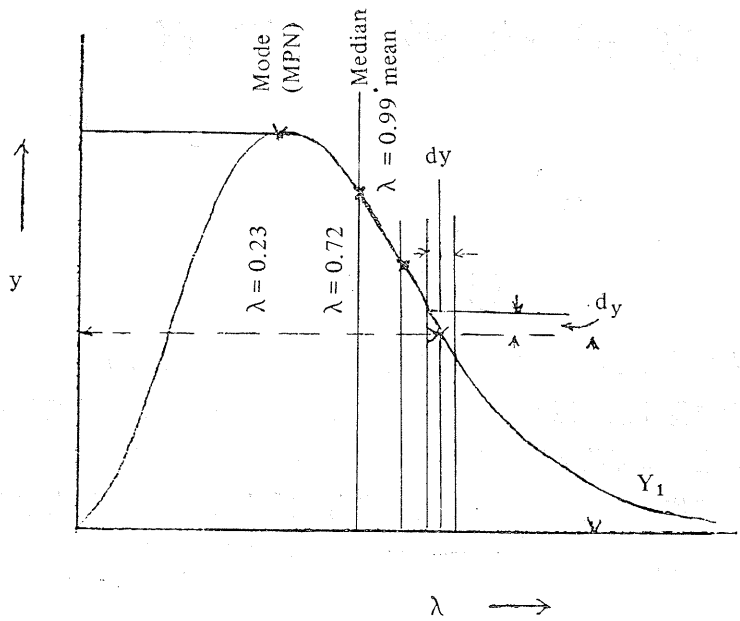
$$\lambda\text{mean}=\left[\int_0^\infty \lambda y\,d\lambda\right]\div\left[\int_0^\infty y\,d\lambda\right] \qquad (2)$$

Also, the median value of $\lambda$ is determined by another relationship:

$$\lambda\text{median}=K$$

$$\text{when } \int_0^K y\,d\lambda=\tfrac{1}{2}\int_0^\infty y\,d\lambda \qquad (3)$$

The positions of these values are indicated in following graphical illustration.




In conclusion, the MPN is a probable value by mathematical reasoning as are the mean value or median value.

## REFERENCE

1. Howe, R. H. L. "Comparative Methods of Waste Treatment and Water Purification", The University of Notre Dame, Visiting Professor Seminar of Environmental Health Engineering, 1973, Unpublished.